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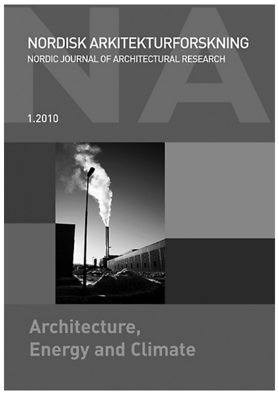

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NORDISK ARKITEKTURFORSKNING – NORDIC JOURNAL OF ARCHITECTURAL RESEARCH

Retracing Khufu's Great Pyramid. The "diamond matrix" and the number 7

Ole Jørgen Bryn

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Abstract:

The geometry of Khufu's Great Pyramid at Giza is even more sophisticated than perceived at first glance. From the first aerial photographs from the 1920`s, it became clear that the pyramid was not set on a square base, rather it has a "diamond" or "star" shaped plan, a fact that is overlooked or avoided in the abundant research literature concerning this pyramid's advanced geometry. Most pyramid building theories deal with only its physical erection, but from an architect's perspective, the first question would not be how to erect it in practical terms but how to develop a system (building grid) enabling the details and precision of the project to be communicated to a workforce of, for example, 10,000 illiterate men.

This paper aims to show how and why the building grid for the last remaining of the seven wonders of the ancient world was developed with the help of the prime number seven. As discussed herein, this was achieved without involving any advanced mathematics. Most importantly, the invention of the building grid represents the separation of the buildings measuring system from the buildings constituent parts.

Keywords:
Khufus Great Pyramid, Building grid, Tollerance, Royal cubit, Seked, Pyramid geometry

Introduction and Background

The term 'true' pyramid is used to differentiate the true ancient Egyptian pyramids from other more common step pyramids, found all over the world. The fact that the Egyptian pyramids are the only true pyramids in the world culminating in an Apex point[1] has two implications:

The first is the need for extreme precision on a grand scale. To aim for a point 146.6 m (280 Rc) up in the sky with only a plumb line and a string, implies the crucial need for numerous points of measure. These points have to be evenly distributed over the face of the pyramid in order for the geometry to be carefully controlled.

The second problem is "the trouble at the top" (Lehner 1997, p 222), viz. how the last blocks of masonry were laid near the apex point.

Before proceeding much further, it is important to note that this paper does not deal with or attempt to solve the "trouble at the top"; this problem has nurtured numerous theories about various ramps and other suggestions (Lehner 1997, p 216). The need for precision is the same, whatever means one applies to get the masonry, to the top; this paper is about lifting precision, not stone.

Historically, the Egyptian pyramids were regarded as piles of stone that needed to be moved one on top of the other. In his otherwise brilliant book, Lehner states: "The masonry of these step pyramids

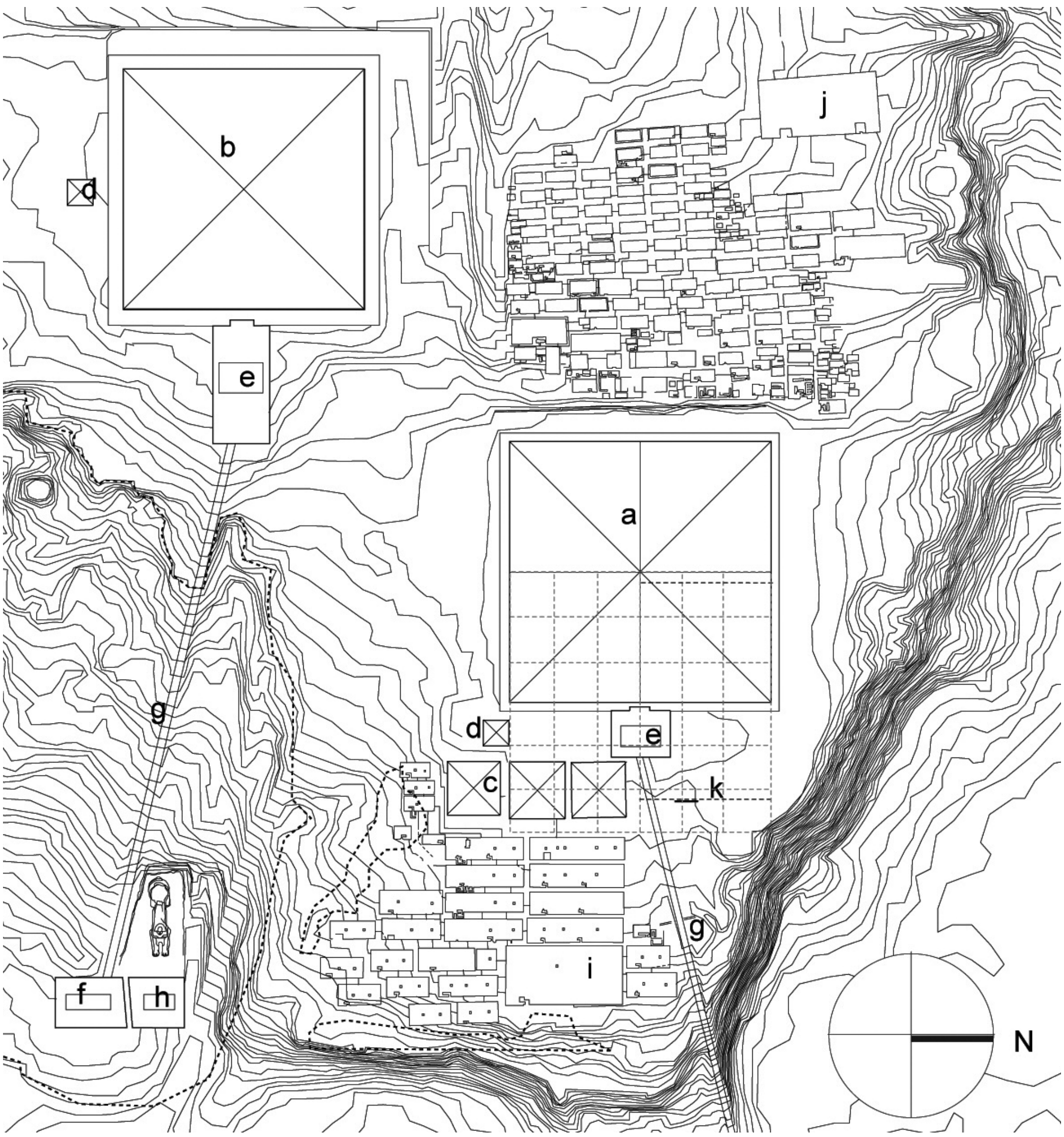

Fig. 1
The Giza necropolis
with the-6 square grid of the master plan.
Map traced from an aggregate of low resolution images

a Khufu's Great Pyramid
b Khafre's Pyramid
c Queen's pyramids
d Cult pyramids
e Mortuary temples
f Valley temples
g Causeways.The 6-square grid
h The Sphinx and temple
i Royal cemetery
j Mastabas
k The 'trial passage'

was clearly too roughly constructed to act as a reference for the outer slope." (Lehner 1997, p221). What is clearly missed here is: One does not necessarily measure from a physical part of a building; one measures more likely from the building's measuring system: the building grid. The invention of the building grid represents the separation of the buildings measuring system from the buildings constituent parts. The purpose of this paper is to show such a system, based on ancient Egyptian units.

Early theories

Khufu's Great Pyramid, GP. 2606-2573 BC[2], placed on the Giza plateau (Fig. 1), is arguably the most studied structure in the history of mankind. Its immense size, sophistication and endurance, has nurtured a great number of theories, some more plausible than others, on how it was built and why its dimensions are what they are.

One of the earliest, Herodotus, states that the: "Egyptian priests told him then that the GP was designed so that the area of each face was equal to the square of its height." (Herodotus 440 BC)

During the nineteenth century, as the GP was measured and re-measured, a number of explorers, Egyptologists and early scientists suggested explanations concerning the shape and size of the GP. For example in 1859 John Taylor (Taylor, J. 1859), an English publisher and Egyptologist, introduced a theory that included π and square roots. In his model, the perimeter was twice π times the height. He went on to claim that the GP was to, "make a record of the measure of the Earth".

In 1863 French Architect Viollet-le-Duc introduced the use of the 3-4-5 triangle to explain the GP's geometry (Viollet le Duc 1863).

The mathematical theory of this triangle was developed 2000 years later by the Greek Scientist Pythagoras, and bears his name. (fig.2)

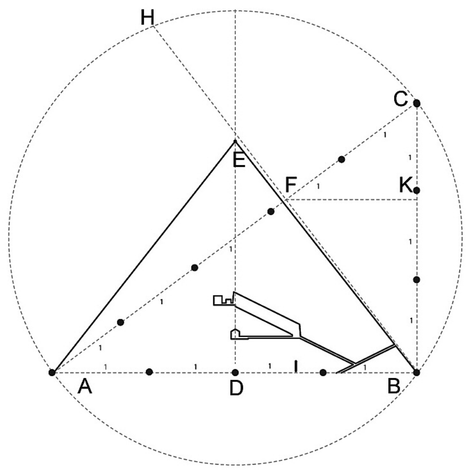

Fig. 2
Viollet le Duc; 3-4-5 triangle

Other theories purported that the exterior slope angle, can be formed with the vesica – pisces[3], That the GP's geometry is an accurate representation of the northern hemisphere, that it incorporates the Greek "Golden section", etc. And while these ideas fell by the wayside for obvious reasons, to this day, none solve the crucial geometrical challenge which is how to build a massive four-sided building whose four sides meet at an apex point. The GP being 146.6 m, (280 royal cubits), high would need a finely tuned, highly accurate, but simple, system to enable, say 10,000 illiterate workers, to reach the apex point.

The building grid

"Point, line, surface and body; of these the point is unique of its kind, and this point has neither breath, nor length, nor depth, wherefore we conclude that it is indivisible and does not occupy space". Leonardo da Vinci. (Ca. AD 15 00)

Most theories on the construction of the GP deal with the logistics of moving roughly 7 million tons of stone, and the physical erection of the monument (Arnold 1991). The natural starting point for the pyramid's architect, however, would be how to develop a system that will enable him or her to communicate both the design and accuracy to an illiterate workforce.

This system, the building grid, was probably necessary in order for the apex point of the pyramid to be established, with the apex point itself being a true grid point. The pyramid ion[4] is the last piece of masonry placed on site, but that point must have been built in to the project documentation during the entire time span of the erection of the monument, one may even claim that the entire building's purpose was to reach that singular point. By focusing on the grid rather than the building itself, it is possible to retrace the architectural drawings showing the internal structure and thereby explain the pyramid's advanced geometry.

Building grids, crucial and indispensible tools in any modern project, provide points of measurement that are not part of the built structure, thereby introducing the tolerance[5] needed to achieve the requisite precision. Today the use of building grids and tolerance is taken for granted when erecting large buildings, yet the separation of the measuring system from the building's constituent parts is not recognized as an invention. Five factors are essential for producing a practical building grid:

- The grid must provide enough points of measure to be practical.
- The numbers in the grid should be divisible, similar, and whole.
- The grid should provide a practical way of labeling geographical positions in the building under construction.
- A true building grid must have a physical structure rising with it, viz. a core
- The grid for a true pyramid must be three-dimensional in order for the apex point to be reached.

A functional building grid would have to be developed from the units of measurement that the Egyptians used; we know about a large number of units in their architectural designs in the 3rd (2727 – 2655bc) and 4th dynasty (2655 – 2484 bc) (Rossi 2003, p61). Here we will focus on the basic set of units used in religious and royal structures in this time span: The Royal cubit.

The Royal cubit

In 1864 Astronomer Royal of Scotland Charles Piazzi Smyth (1819-1900) went to Giza and made accurate measurements, on the GP, He then published the first work on the units of measurements used on the GP (Piazzi - Smyth, 1867). His theory tried to link "the pyramid inch" to the British inch. He was inspired by, and collaborated with John Taylor to prevent the introduction of the metric system in Britain.

Needless to say, these ideas also fell to the wayside for the simple reason that he did not take into consideration the well established historical fact that the ancient Egyptian used the royal cubit, and it was highly unlikely that the GP builders would use other measures. The Egyptian Royal cubit represented the fore arm of the pharaoh and was divided in 7 palms. The latter was divided in four fingers or digits. (Arnold 1994, P 61) (Fig. 3)

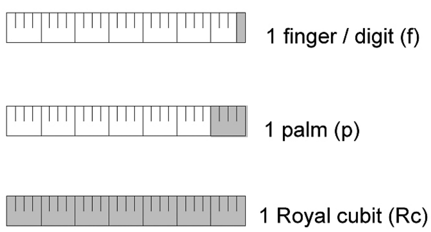

Fig 3
Egyptian units

In the early 1880s, W.H.F Petrie traveled to Giza; he was the first truly modern archeologist to study the GP. His measurements on the Giza plateau in general, and on the GP in particular, are so accurate, that they are still the main source of measurements used to this day. (Petrie 1888). Based on his measurements, it is now established that the exact length of an Egyptian royal cubit, Rc, as used for the GP, was 52.355 cm.

The *SEKED*

The Egyptians used the 'rise and run' method to measure angles (Lehner 1997, p 218).
A *seked* is the horizontal distance run for every 1 Rc rise (7 palms).
The *seked* of the slope of Khufu's GP is 5 palms, 2 fingers run, for every one Rc rise. Likewise, the *seked* of the slightly steeper Khafre pyramid is 5 palms, 1 finger. The *seked* was used to define most pyramid slopes in the Old Kingdom (Rossi 2003, appendix).

Researchers have looked in vain for the tools used at the building site to implement this angle of slope. This paper wants to show that the *seked* was most probably used on the drawing board and not at the building site. This specific rise and run had to be translated in to other means of precision when transferred to a building site. The only precise "tool" available to the Egyptians on this scale, was gravity. It has always worked. Herein it is shown how it was possible for the GP, and others, to reach the elusive apex point using nothing more sophisticated than a plumb line and a string.

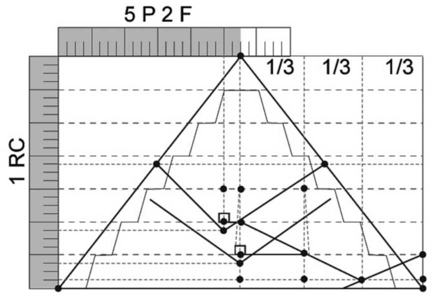

Fig 4
The GP drawn at 1:280

The Number Seven

A unit divided in 7 parts (1 Rc = 7 p) is not suited for a building grid, 7 being a prime number. To solve this problem the Egyptians divided the pyramid in 7, which produced a 6-stepped core structure with the seventh step replaced by the apex point (Fig. 4). As noted above, the GP is 280 Rc high with a *seked* of 5 p, 2f (S=5,5), which results in a baseline of 440 Rc. To produce a 3 dimensional building grid for this 6 stepped structure, one needs to divide 280 Rc by 7. Likewise you need to divide half the base line, 220 Rc, in 7. Since the Rc are divided into 7 p one only needs to change the units from Rc to p; the number is the same. (Note that this method works whatever height is chosen).

At 220 Rc half of the baseline, now equals a grid of 220 p in plan. The horizontal distance from each *mastaba's*[6] base to the surface of the pyramid (Fig. 5) then equals 220 p, and the distance vertically equals 280 p. This way of changing the unit so that the building grid becomes easily divisible (1p = 4 f) results in an abundance of measuring points evenly distributed throughout the pyramid's volume and hence its face (Fig. 5). This made it possible to ultimately establish the geometry of the GP, culminating in the apex point.

Figure 4 shows the most probable first drawing of the GP at a scale of 1:280. This drawing would be 1 Rc high. If the cubit stick, with the divisions of palms and fingers represent the height of the pyramid, then most of the layout of Khufu's GP can be drawn using only the markings for fingers and palms on the stick.

The *seked* is, of course, natural to use to determine the slope at this scale, because the drawing is 1 Rc high.

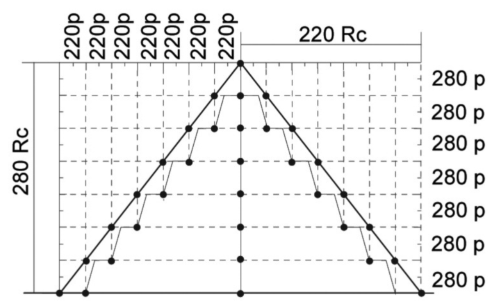

Fig 5
The GP's building grid

Probably one of the strongest evidence for the division of the pyramid into 7 parts is the familiar section of Khufu's GP. As shown in Fig. 6, now the locations of the two above ground chambers and the grand gallery make perfect sense. The Queen's chamber[7] (b in Fig. 6) is placed precisely on top of the first *mastaba*, the King's chamber[8] (a in Fig. 6) is on the second *mastaba*, and the Great Gallery[9] (c in Fig. 6) forms a ramp between the first and second *mastabas*.

Some theories on the interior layout of the GP, point out the fact that the level of the King's burial chamber is where the area of the pyramid in plan is half of the area at the base (Romer, 2007 p2). This is most probably a consequence of dividing the pyramid into 7, rather than a cause; the area of any pyramid 2/7 up is 0.51 the area at the base.

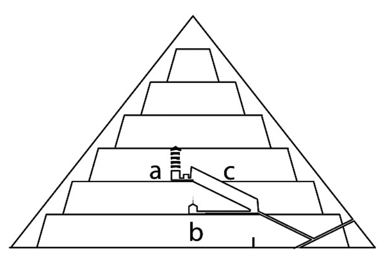

Fig 6
The GP's interior layout

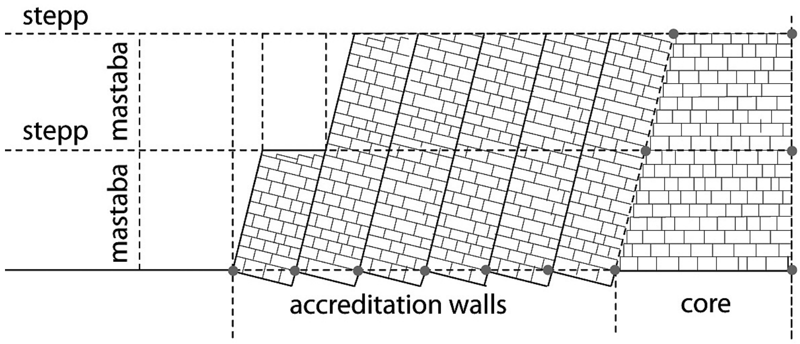

Fig 7
3rd. dynasty stepped pyramid

The core

The need for a structure to rise with the grid is obvious. When putting in place the curtain wall on a skyscraper, the precision is already (hopefully) in place with in the concrete and steel core.

A step pyramid is ideal to raise a grid.
We know from the earliest 3rd dynasty step pyramids in Egypt that they were built with accreditation walls or layers. (Arnold 1991, p 160). In those early pyramids the accreditation layers were tilted inwards at an angle. The tilting was probably done to support temporary ramps from step to step and/or to provide structural stability.

Each part of the pyramid between two steps is referred to as a *mastaba*. Each accreditation wall or layer formed one step of the step pyramid and defined the base of the next mastaba by reducing the height of the wall layer by layer. (Fig. 7) (Arnold 1991, p 159)

To this day the interior or core of Khufu's GP is hidden from view by the preserved masonry. Most archeologists, however, believe that the GP is comprised of horizontally laid masonry with the same height of the masonry visible on the surface today. This paper will try to show that the core of Khufu's great pyramid is most probably a step pyramid.

This step pyramid was used to transfer the pyramids precision system, the building grid vertically, and that the blocks seen to day are backing blocks[10], that do not necessarily align with the underlaying *mastaba's* (Fig. 8). This paper claims that the accreditation walls forming the stepped core of a true pyramid are made with horizontal laid masonry. (One exception is the Meidum pyramid which started out as a step pyramid with tilted masonry.) The accreditation wall's that formed the face of each *mastaba* was kept at the same tilted slope as the accreditation walls of the old type, step pyramids. (Compare Figs. 7 and 8). This was done to support temporary ramps from step to step, and not to let the core break through the face of the pyramid. The existence of such temporary ramps was first suggested as a construction method for step pyramids by Hoelsher in the book *Das Grabdenkmal des Konigs Cephren, frontispiece.* (Somer Clarke and R. Engelbach 1930, p 120,)

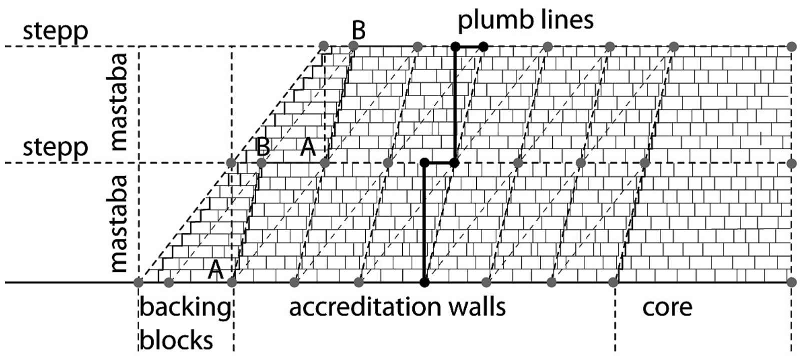

Fig 8
4th. dynasty true pyramid with stepped core

The Egyptians most probably used the base of each mastaba for measuring the face of the finished pyramid. It is quite possible that it was the mastabas that were first carefully geographically aligned North-South, and not the baseline we see today. Note that the grid points used to measure out to the face of the pyramid would have been placed at the base (point A in Fig. 8), and not the top (point B in Fig. 8), of each *mastaba*.

By erecting walls with horizontally laid masonry, they could introduce tolerance between the accreditation layers, thereby transferring the building grid vertically from *mastaba* to *mastaba* with a plumb line. (Fig.8).The outlines of each accreditation wall were carefully measured out on top of each *mastaba*. Those lines and point could now be adjusted using 3-4-5 triangle and the diagonal. Those lines represented the building grid.

The six-square grid

Square grids, in general, were used by the Egyptians for the layout of temples and religious compounds (Rossi, 2003, p 122). Romer's 6-square grid theory (Romer 2007, p 65) is based on the over-all 6-square grid of the master plan (Fig. 1). The layout of Khufu's Queen's pyramids, satellite pyramid, and the mortuary temple (Lehner, 1985) (Fig. 1) is good evidence for its correctness. In plan, this grid works, not only for the master plan, but as importantly for the interior layout of the pyramid, because the latter was part of the former. Romer's grid, however, was probably not used as a grid for the construction of the pyramid itself for three reasons. First, the grid, when transferred to Khufu's GP (Fig. 9), lacks any relation to an existing built structure (core); it has too few grid points, and it cannot be divided into smaller fractions. Thus, as a practical building grid it is not usable. In other words, the master plan grid cannot be directly converted to a 3-D construction grid. However, as discussed in the next section, this master plan grid may have influenced the exterior of the pyramid in a rather unexpected way.

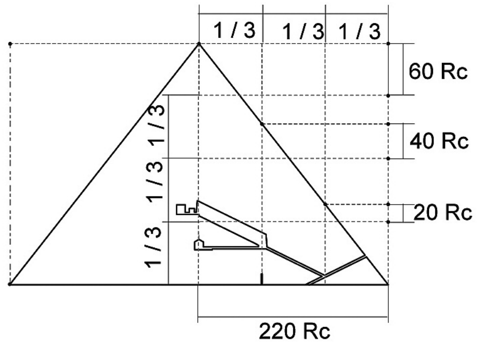

Fig 9
The 6 square grid used to explain the geometry of the GP

The 'diamond matrix'

Aerial photographs, taken in the 1920s, reveal that Khufu's GP was not set on a square base. (Fig. 10[11])The centre line on each face is moved inwards by a short distance at the base, yet the deviation is so small that it cannot be detected at ground level given the present-day uneven surfaces. Most researchers seem to have overlooked or ignored this fact, possibly because it is difficult to explain the level of precision involved. This author has not found one serious explanation of this fact.

Based on the ideas presented above, a straightforward explanation exists. The "diamond matrix" results naturally, if the centre lines on each face are moved back towards the pyramid's center, by 1 Rc or 7 p, at the base of the first *mastaba* (Fig. 11).

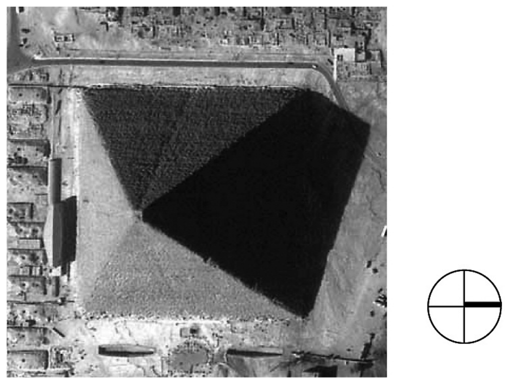

Fig 10
Satellite image the GP
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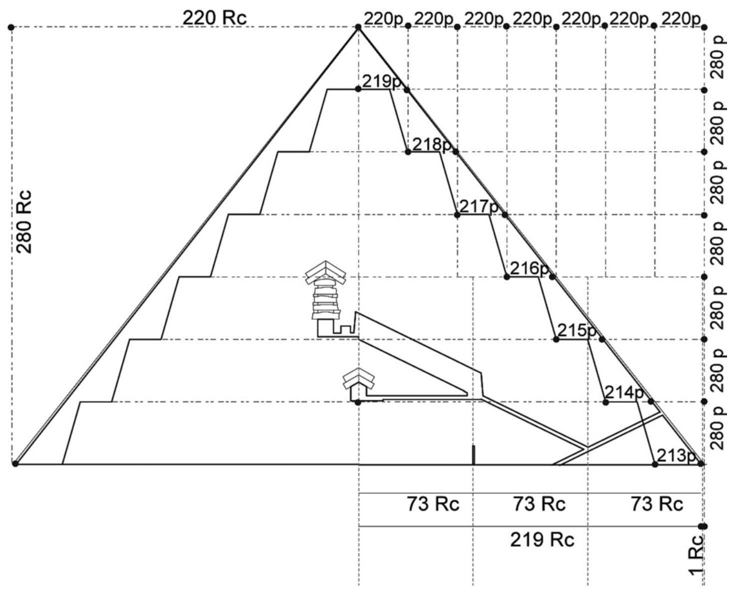
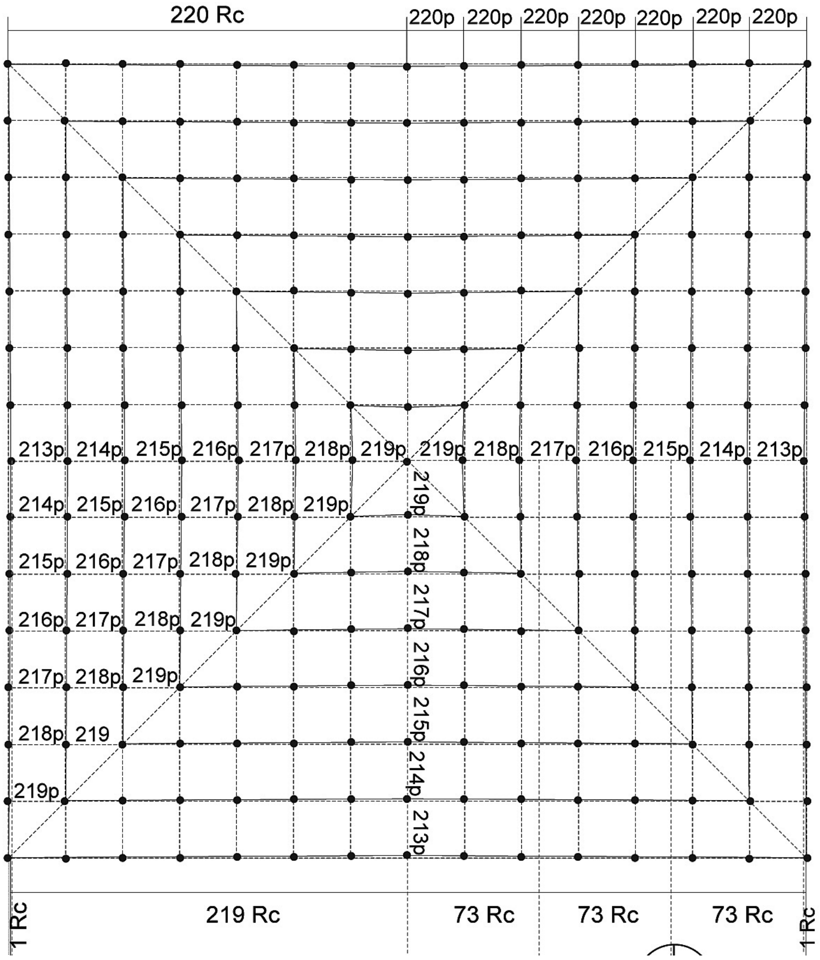
The logic in subtracting 1 Rc from the baseline relates to the six-square grid. The interior layout, based on half of the base divided by three, does not produce whole numbers when half of the base is 220 Rc. But if 1 Rc is subtracted at the centre, where the interior is determined, the result is 219 Rc. which produces exactly 73 Rc when divided by 3. Since the pyramid grid predicts a distance from each *mastaba's* base to the surface of the pyramid of 220 p, removing 7 p at the base of the pyramid and then adding one p for each step creates the "diamond matrix" in section (Fig. 11).

Likewise, removing 7 p at the centre of the base and then adding one p for each gridline in plan produce the"diamond matrix" in plan (Fig. 12). If that is the case, then the 6-square grid of the master plan clearly influenced the main building grid, and thereby the geometry of the GP.

Fig 11
The "Diamond matrix" in section





The motivation behind the diamond matrix is unknown, but it could have been religious or aesthetical. It would undoubtedly have been easier to make a straight *true* pyramid. Nonetheless, Khufu's GPs diamond matrix clearly testifies of the invention and use of a versatile building grid, with a very high degree of flexibility, without involving π , the golden section or any other form of advanced mathematics.

Fig 12
The "Diamond matrix" in plan

Summary and Conclusions:

The Apex point of a True pyramid could not have been reached without the precision and tolerance embedded in a simple and redundant 3-dimensional grid with the precision system separated from the buildings constituent parts. Since this idea is taken for granted in the architectural profession, and among every brick layer and mason, what is most surprising is that it has never been applied to the most widely discussed building in the history of mankind.

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The Giza cemetery from the Abu Rawash pyramid.
Foto: Håvard Houen

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NOTES

[1]Apex point is the summit of the true pyramid Grid

[2]The timeline is from Sitek, D. Ancient Egypt History & Chronology. www.narmer.pl/indexen.htm (accessed 2008).

[3]The *Vesica piscis* is a shape which is the intersection of two circles with the same radius, intersecting in such a way that the center of each circle lies on the circumference of the other. From Wikipedia accessed 2008

[4]Pyramid ion is the physical summit of the pyramid (the last block representing the Apex point of the pyramid) (Arnold D 1994, P 189)

[5]Tolerance can be explained as follows: The difference of 30 Rc being made out of 3 walls of 10 Rc leaning on each other as opposed to 30 Rc being 3 walls put at 10 Rc interval.

[6]The *mastaba* (Arabic; bench) was the basic form of funeral architecture of the Old Kingdom (2727 – 2175 BC) in Egypt. The first Step pyramids are sometimes referred to as *Mastaba* pyramids.

[7]The Queen's chamber has its name from an Arabic interpretation of this chamber, resembling a woman's chamber. It is unlikely, though, that this was intended as a burial chamber for a queen.

[8]The Kings chamber was the Pharaohs burial chamber

[9]The grand gallery is a unique peace of interior design both in size and statue. Measuring: length: 47.8 m height: 8.5...8.7 m slope: ca. Raise 1 run 2

[10]The term "Backing blocks" is used in this paper for the blocks filling out the steps on a stepped core of a true pyramid. Various names are used on this part of the Masonry.

[11]Image provided courtesy of Digital Globe and Satellite Imaging Corporation

LITERATURE

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